D1STRIBUTIONS OF THE INTERPLANETARY MAGNETIC FIELD REVISITED

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Abstract. The adequacy of the power spectrum to characterize the variations of a parameter depends on whether or not the parameter has a Gaussian distribution. We here perform very simple tests of Gaussianity on the distributions of the magnitudes of the interplanetary magnetic field, and on the distributions of the components i.e. wc find the first four cumulants of the distributions (mean, variance, skewness and kurtosis). We find that the traditional distributions of the one hour averaged magnitude are not distributed normally or log-normally as has often been assumed and the one hour averaged z component is found to have a non-zero kurtosis. Thus the power spectrum is insufficient to completely characterize these variations and polyspectra are needed. We have isolated variations in the 1/f frequency region of the spectrum and show that the distributions of the magnitudes have non-zero skewness and kurtosis, the magnitudes are not distributed log-normally, and the distributions of the components have non-zero kurtosis. Thus higher order spectra are again needed for a full characterization. The origin of the 1/f spectrum is discussed and a model for the production of the corresponding magnetic fluctuations is suggested.

Introduction

Recent progress in the study of the fluctuations of the magnetic field in space have re-emphasized the importance of the distributions of the observed magnetic field parameters. In particular, if the fluctuations of a quantity have a Gaussian distribution then the power spectrum is sufficient to characterize these fluctuations. However, if a parameter, such as the magnetic field magnitude, is not distributed normally, higher order statistical moments and higher order spectra arc required to completely describe the fluctuations (Brillinger and Rosenblatt, 1967, Priestley,1991). Although the power spectrum of the interplanetary field variations have been studied extensively (see for example review by Roberts and Goldstein, 1991) the question of the adequacy of the power spectrum to completely characterize the fluctuations has not been addressed. Here wc carry out such a study by finding the statistical moments (cumulants) of the distributions of the magnetic field fluctuations. The data used in this study arc from the OMNITAPE supplied by the NSSDC (National Space Science Data Center). This tape contains a compilation of data from various spacecraft that were observing the interplanetary medium in the vicinity of the Earth and is the most complete data set available for this stud y. The time interval covered in our study is from January 1,1973 to July 12,1991.

The first four moments (cumulants) of the distributions arc defined as follows

$$k_1 \equiv \langle x \rangle = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \textit{mean}$$

$$k_2 \equiv \sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \langle x \rangle)^2 \qquad \text{variance}$$

$$k_3 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i - \langle x \rangle}{\sigma}\right)^3 \qquad \textit{skewness}$$

$$k_4 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i - \langle x \rangle}{\sigma}\right)^4 - 3 \qquad \textit{kurtosis}$$

where <> indicates averaging, xi arc the observed values of the variable, and N is the number of data points. The skewness is a measure of the lack of symmetry of the distribution. A positive (negative) number indicates a higher number of large (small) values of the parameter than would be expected for a Gaussian distribution. The kurtosis is a measure of the flatness (negative value of k_4) or peakedness (positive value of k_4) of the distribution relative to a normal distribution with the same mean and standard deviation. Note that while the mean and standard deviation have the same dimensions as the measured quantities, the skewness and kurtosis arc dimensionless.

For a normal (Gaussian) distribution all cumulants higher than the second arc zero,

$$k_3, k_4, \ldots, k_n = 0.$$

Observations

One Hour Averaged Data

Traditionally the interplanetary magnetic field distribution has been presented as a distribution of hourly averages. Early measurements showed clearly that the one hour averages of the magnitude of the field were not distributed normally. There was an obvious high field tail that has been attributed to solar wind associated with coronal mass ejections and with the compressions due to the interaction of solar wind regions of different speeds (Hirshberg, 1969, Neugebauer, 1991). The solar cycle variation of the mean has also been studied (cf. Hirshberg, 1969, Winterhalter et al., 1990). No systematic studies have been made of the solar cycle variations of the higher moments (variance, skewness and kurtosis) of these distributions. Burlaga and King (1979) suggested that the distributions of the logarithms of the field magnitudes and components could be well represented as normally distributed. In accordance with this suggestion the mean of the distribution of the log of the onc hour average data has been widely used, for example, in studies of the solar cycle changes in the intensity of the field (Slavin and Smith, 1983). However no studies have been reported in which the log-normality of the distributions has been formally tested. If the distributions are truly log-normal then the mean and standard variation of the log distributions should completely characterize thcm and they should have no higher cummulants such as the skewness or the kurtosis.

In this section we study the solar cycle dependence of the moments of the distributions of the hourly averaged magnetic field and present, for first time, the solar cycle variation of the higher moments.

Wc also test the hypothesis that the hourly averaged intensities of the magnetic field arc distributed log-normally. Wc find that thc distribution of logarithms is non-Gaussian and study the solar cyclc variation of the statistical momcnts.

A typical distribution of the onc hour averages of the magnetic field magnitude is shown in Figure la. The fitted curve is a Gaussian that has the same mean and standard variation as the observed distribution. The fit is obviously very poor. The high values of the skewness and the positive value of the kurtosis reflect the largc number of high intensity values. This distribution has more large amplitude deviations than a Gaussian distribution with the same mean and standard deviation.

The four cumulants for the magnitude B have been calculated for the years 1973 to 1990 and their solar cycle variations are shown in Figure 2, pancl a and b. The solar cycle variations of the mean has been discussed in earlier studies (see for example Slavin and Smith, 1983). Figure 2a shows that sigma and the mean follow each other closely throughout the solar cycle. The skewness and kurtosis (2b) are both non-zero and positive. The kurtosis shows a strong solar cycle variation which differs from that of the mean. The solar cycle dcpcndcncc of the skewness is less obvious but it appears to bc larger than average in those years when the kurtosis is large. Note that the skewness and kurtosis maximize in 1979, a sunspot maximum year, whereas the mcan magnitude maximizes three years later, in 1982.

Also of interest are the statistical moments of the distributions of the logarithms of the magnitude of B, since B has been characterized as log-normal (King and Burlaga,1979). Figure lb shows a ‘typical distribution of this quantity. The Gaussian is now a much better fit to the data. However the distribution still exhibits skewness and kurtosis. In order to establish the reality of an observed skewness or kurtosis some statistical considerations must bc taken into account. If a finite sample is sclcctcd from an infinite Gaussianly distributed parent population then the momcnts of the sample will have observed values which are not the same as the moments of thc parent distribution. If many such samples arc drawn from the parent distribution then we will have a distribution of the moments of the samples. Now the observed values of the magnetic field can bc considered to bc a sample drawn from a parent distribution which may or may not bc Gaussian. Wc want to find thc probability that wc would have gotten the observed moments if the parent distribution had been Gaussian. If this probability is small then the probability that the parent distribution was in fact Gaussian is small. The probability that any particular value of the higher moments would bc observed in a single sample drawn from a parent population with a Gaussian distribution can be found from the statistical properties of the distributions of the moments of samples, If wc assume the parent distribution was normally distributed than the variance of thc distribution of the skewness is estimated to be square root of 6/N, where N is the number of objects in the sample, which is assumed to bc large. The variance of the kurtosis is estimated to bc the square root of 24/N (see for example Press et al., 1986).

Figure 2 panel c and d shows the solar cycle variation of the moments of the distribution of log B. The mcan shows the wcll known solar cycle variation (King, 1979; Slavin and Smith, 1983). The skewness is usually positive, although small negative values occur. Using thc typical sample size for this data, we estimate the variance of the skewness to bc about 0.03. Although a few values of the observed skewness fall within this uncertainty most of thc values are many times that value, indicating that the skewness is rcal and the distributions are samples of a non-Gaussian parent population. There also appears to be a tendency for the skewness to be smallest around sunspot minimum and positive when the sunspot number is higher. The case for thc non-Gaussian distribution is made even more clearly by considering the kurtosis. If each of the yearly samples had been drawn from a Gaussian parent population then negative values of the kurtosis should bc expected as frcqucntly as positive values. Here the observed values arc much larger than the uncertainty of about 0.06 and, morc importantly, all positive, There can bc no doubt that the kurtosis is non-vanishing and that therefore the distribution of the hourly average magnitudes is not log-normal,

A typical distribution of the one hour averaged z component is shown in Figure 1c. If the magnctic field were a Parker field with no disturbances than there would bc no z component in the ecliptic plane. The distribution shown has a mean which is essentially zero; however it has a substantial skewness and kurtosis. The solar cycle dependencies for the z component are shown in Figure 2, panels c and f. Sigma has a strong solar cycle variation that follows that of the magnitude. The skewness varies little with the solar cycle and has a mean value of only 0.086 averaged over 17 years, indicating that the power in the bispectrum is very small for this data set. However the kurtosis is substantial throughout the period, indicating the importance of the trispectra for this data set.

Three hour averages: the 1/f spectral range

The variations in the observed magnetic field have different physical causes, depending on the size of structure involved or the wave length of the disturbance. Matthaeus and Goldstein (1986) have studied the spectrum of the magnetic field intensity at 1 AU and found that it could be broken into three frequency regions, a very low frequency part (below 2.7 X10-6 Hz or about 100 hours), a 1 /f spectrum at higher frequencies, and a Kolmogorov type spectrum at frequencies higher than 8.0x10-5 Hz, corresponding to periods of about 3 hours. The boundaries of these frequency intervals approximately correspond to a transition time needed to cover the distance from the Sun to 1 AU (100 hrs.), and to the inverse correlation time for the magnetic field in the solar wind (about 3.4 hrs). The different spectral behavior in each of these regions shows that the physics involved in producing the variations may also be different. The distribution of the hourly averages discussed above depends on the behavior of all of the variations of frequencies longer than 1 cycle/hour: the very low frequency region (including the changes in B across high speed streams), the 1/f region, and some of the Kolmogorov variations. For purposes of theoretical understanding it is important to study the distributions in each frequency range separately. In the remainder of this paper we study the 1/f region.

To eliminate variations coming from the Kolmogorov range we used three hour averages of the data since averaging acts as a low pass filter. To eliminate the contribution of the largest scale structures we subtracted a 100 hour moving average from the components of the 3 hour data and constructed the magnitude of variations from those band passed components. In this way we isolated the variations in the 1/f range. The statistical analysis of these band passed data from 1973 to 1991 was carried out in the same manner as it had been for the one hour averaged data.

The distribution of the filtered magnitude, B, shown in Figure 3a, is very far from normal. This is an important result because it indicates that the power spectrum of the magnetic field magnitude in this frequency region is insufficient to completely characterize the variations of B. The higher order spectra arc also required for a complete characterization. The solar cycle variation of the moments of the distribution of the band passed field magnitude are shown in Figure 4 panels a and b. The solar cycle variation of the mean and sigma arc clearly seen and arc alike. Since sunspot maximum occurred in 1979-1980 and in 1991, it is evident that the variations in this quantity are not in phase with the sunspot cycle, as was well known for the unfiltered data. The skewness is always positive and far larger than its expected uncertainty of 0.05. Likewise the kurtosis is large, positive and well outside the uncertainty of 0.1. Thus the distributions of the magnitudes in the 1/f region arc non-Gaussian.

The distribution of log B shown in Figure 3b was found to be close to normal (Matthaeus and Goldstein, 1986) but it differs from normal as shown by the non-zero skewness and kurtosis. The solar cycle variations of the mean and sigma (Figures 4c, 4d) differ only slightly from the time dependencies presented in Figures 2c, 2d for the non-filtered data. The most interesting differences are in the skewness and kurtosis. The average skewness is -0.3 and all the values arc negative. The kurtosis is consistently positive.

Two types of magnetic field power spectra appear in the literature, the spectrum of the magnitude of B and the spectrum of the invariant trace of the components of B. In order to test the sufficiency of the power spectrum to characterize the invariant trace spectra in the 1/f region, we must examine the distributions of the components. The z component of the field is taken as an example and its distribution in 1973 arc shown in Figures 3c. The solar cycle variation for the z component is shown in Figure 4, panels c and f. As expected from symmetry arguments, the mean and skewness are zero. However, the kurtosis is large and positive, showing that polyspectra are required for a complete characterization of the variations.

On the Origin of the 1/f Magnetic Noise

To explain the origin of the 1/f spectrum Matthaeus and Goldstein (1986) developed an interesting model of multiplicative process which starts with a random distribution of relatively small magnetic structures in the low solar atmosphere and develops in the region below the transonic point (where the solar wind becomes supersonic). In this region magnetic structures of different sizes merge and reconnect to form magnetic structures of progressively larger and larger sizes. After many steps the sizes of the magnetic structures, as a multiplicative random variable, are expected to be log-normally distributed. The 1/f spectrum is produced near the Sun, and then the spectrum, in a frozen-in form, is transferred by the solar wind through the heliosphere. In this picture the 1/f spectrum would be a feature observed in the solar wind only, not a feature observed in other systems in which magnetohydrodynamic turbulence is developing.

Small-scale turbulence can be expected to have an effect on the interplanetary propagation of the 1 /f spectrum. Such turbulence is observed in the solar wind in the form of an inertial, Kolmogorov type spectrum at wavelengths smaller than the 1/f range. The collective effect of this small scale turbulence can be described as an effective diffusivity which results in the exponential decay of every mode of the 1/f spectrum, The characteristic time of this decay depends (quadratically) on the wavelength of the mode so that one can expect that 1/f modes with the smallest wavelengths will decay first, i.e. at smaller heliocentric distances. Eventually the entire 1/f spectrum will decay. Whether or not this effect will be observable depends on the time scale for the decay versus the time scale for the production of the Kolmogorov variations in the same frequency range.

We would like to discuss here another possible mechanism of creating the 1/f spectrum. This mechanism essentially involves the small-scale turbulence and the large-scale magnetic field not considered in the above scenario. In the scenario discussed below wc do not assume that the 1/f spectrum is created below the transonic point or by processes specific to the Sun. Instead the 1/f spectrum is actively created at all heliocentric distances provided only that there is a large-scale magnetic field and small-scale turbulence.

The process under consideration produces magnetic fluctuations by simple kinematic deformations of the large-scale field with a help of velocity fluctuations. To do this only a fcw characteristic times $t_c = L_c/v_c$ arc needed, where v_c is the characteristic velocity at the scale L_c and is of the order of v_A, Such a mechanism was used earlier to derive a 1/k spectrum for the galactic magnetic field (Ruzmaikin and Shukurov, 1982). Recent numerical simulations of magnetic fluctuations produced by a random velocity field in a large-scale magnetic field also show the 1/k spectrum (Brandenburg et al., 1993). (In the solar wind a spatial 1/k spectrum will actually bc observed as a 1/f spectrum since $f=kV_{sw}$, where V_{sw} is the solar wind speed,) In this picture the 1/k spectrum appears at intermediate values of k between the non-universal spectrum describing the large scale (k small) variations input into the system and the inertial spectrum of the Kolmogorov type at higher k.

Although there is as yet no elaborated theory of the process, wc can present some order of magnitude estimates. Consider a well conducting plasma in which a large-scale magnetic field, B, is disturbed by chaotic hydrodynamic motions of the velocity, v. We may estimate the magnetic fluctuations, b, produced in this way by using Ohm's law $E=\eta j$. Here the electric field $E \propto v \times B$, the current $j \propto \mathrm{curl}\, b$, and the turbulent resistivity η is determined by the inertial spectrum with higher wave numbers in the Kolmogorov range. Thus, rewriting Ohm's law in terms of the Fourier amplitudes, $v_k B \propto \eta k b_k$. It follows that the spectral densities of the magnetic fluctuations, M(k), and velocity fluctuations, E(k), arc related as

$$M(k) \propto E(k)\,(\eta\, k)^{-2} B^2.$$

The substitution $\eta^2 \propto k^{-2} v_k^2 \propto k^{-1} E(k)$ gives $M(k) \propto B^2 k^{-1}$.

We speculate that 1/k spectra appear as part of the transition from quasi-regular magnetic structures to chaotic magnetic fluctuations during the development of turbulence. In the case of the solar wind the quasi-regular magnetic structures emanate from the Sun and the chaotic fluctuations are the Kolmogorov variations (around a mean value) as observed at 0,3 AU and beyond, The number of characteristic times, t_c, that have transpired increases slowly with heliocentric distance. For example as $L_c/v_c \propto L_c^{2/3}$ for the Kolmogorov spectrum compared with the passage time for the same scale ($L_c/v_{SW} \propto L_c$). Therefore one can expect that the boundary between the Kolmogorov type spectrum and the 1/k spectrum moves towards the larger scales (low frequencies) with distance from the sun. The observations indeed show such shrinking of the 1/f spectral region with heliocentric distance (Goldstein et al., 1984).

Summary and Discussion

This paper is based on the recognition of the fact that if the distribution of a parameter is not Gaussian then the power spectrum of that parameter does not completely characterize the quantity. Instead, spectra of the higher order moments (polyspectra) are also required. This is a corollary to the fact that for a Gaussian the distribution is completely described by the first two moments of the distribution (the mean and sigma). The higher order moments are all zero (for odd moments) or can be expressed in terms of the mean and variance (for even moments). It then becomes important to test the Gaussian nature of the distributions of the quantities for which the power spectra appear in the literature.

As is well known, the distributions of the one hour averaged interplanetary field magnitude are non-Gaussian, however, they have been said to be log-normal. We found that these data were not distributed log-normally as had been stated earlier in the literature, The distribution of the components of the one hour averaged magnetic field was also shown to be non-Gaussian.

In order to isolate variations having separate physical causes wc used a bandpass filter to isolate variations in the 1/f frequency range. The distribution of thc magnitudes and components of these variations was again found not to be normally distributed, showing that the power spectrum was insufficient to characterize them.

The origin of 1/f variations first discussed by Matthacus and Goldstein (1986) was reconsidered. An alternate explanation was suggested in which these variations arose from thc deformation of the magnetic field by chaotic fluid turbulence in the solar wind.

Our results have important implications for both theory and analysis of observations. We have shown that the interplanetary ficld variations in the 1/f region of the spectrum have non-Gaussian distributions. In this case higher moments form irreducible independent quantities. Thus spectra of the higher order correlation functions (polyspcctra) are required to completely characterize these variations,

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References

Brandenburg, A., A. Nordlund. A. A. Ruzmaikin, and R. Stein, Numerical simulations of dynamo action in the Sun, in Research *Trends in Plasma Astrophysics,* International Topical Conference, La Jolla, California, November 1993.

Brillingcr, D. R., and Roscnblatt, M., 1967, Computation and interpretation of k-th order spectra, in *Spectral analysis of time series,* ed. by B. Harris, John Wiley & Sons, Inc., New-York, London, pp. 189-232.

Burlaga, L. F., and J. H. King, Intense interplanetary magnetic fields observed by geocentric spacecraft during 1963-1975, *J. Geophys. Res.*, 84,6633, 1979.

Goldstein, M. L., L, F. Burlaga, and W. H. Matthaeus, Power spectral signatures of interplanetary corotating and transient flows, *J. Geophys. Res., 89,3747,* 1984.

Hirshberg, J. The interplanetary field during the rising part of the solar cycle, *J. Geophys. Res., 74, 5314,1969.*

King, J. H. Solar cycle variations of IMF intensity, *J. Geophys. Res., 84, 1979.*

Matthaeus, W. H., and Goldstein, M. L., Low-frequency 1/f noise in the interplanetary magnetic field, *Phys. Rev, Let f., 57, 495-498, 1986.*

Neugebauer, M., The quasi-stationary and transient states of the solar wind, *Science,* 252,404-409,1991.

Press. W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes,* Cambridge Univ. Press, Cambridge, 1986.

Priestley, M. B. 1991, *Non-linear and Non-stationary Time Series Analysis,* Academic Press, Harcourt Brace Jovanovich, Publishers, London, San Diego.

Roberts, A., and M. Goldstein, Turbulence and waves in the solar wind, *Reviews of Geophysics,* 932,1991

Ruzmaikin, A. A., and A. M. Shukurov, Spectrum of the galactic magnetic field, *Astrophys. Space Sci., 82,397,1982.*

Slavin, J. A., and E. J. Smith, Solar cycle variations in the interplanetary magnetic field, in *Solar Wind Five,* NASA Conference Publication 2280, pp. 323-331,1983.

Winterhalter, D., E. J. Smith, J. H. Wolfe, and J. A, Slavin, Spatial gradients in the heliospheric magnetic field: Pioneer 11 observations bet wcen 1 AU and 24 AU, and over solar cycle 21,], *Geophys. Res.,* 95, 1-11, 1990.

Figure Captions

Fig. 1. A distributions of the one hour averaged magnetic field in 1973. The x and z components arc shown in panels a and b, magnitude is shown in panel c, log magnitude in panel d. The solid lines show normal distributions with the same mean and variance as the sample.

Fig. 2. The solar cycle variation of thc statistical moments of the one hour distributions: (a,c,c) mean value and variance; (b,d,f) skewness and kurtosis. The panel numbers have the same meaning as in Fig. 1. Note that none of the quantities is Gaussian.

Fig. 3. The same as Figure 1 but for the three hour averaged quantities for the band passed data.

Fig. 4. The same as Figure 2 but for the three hour averaged quantities for the band passed data. Note that none of the quantities is Gaussian.

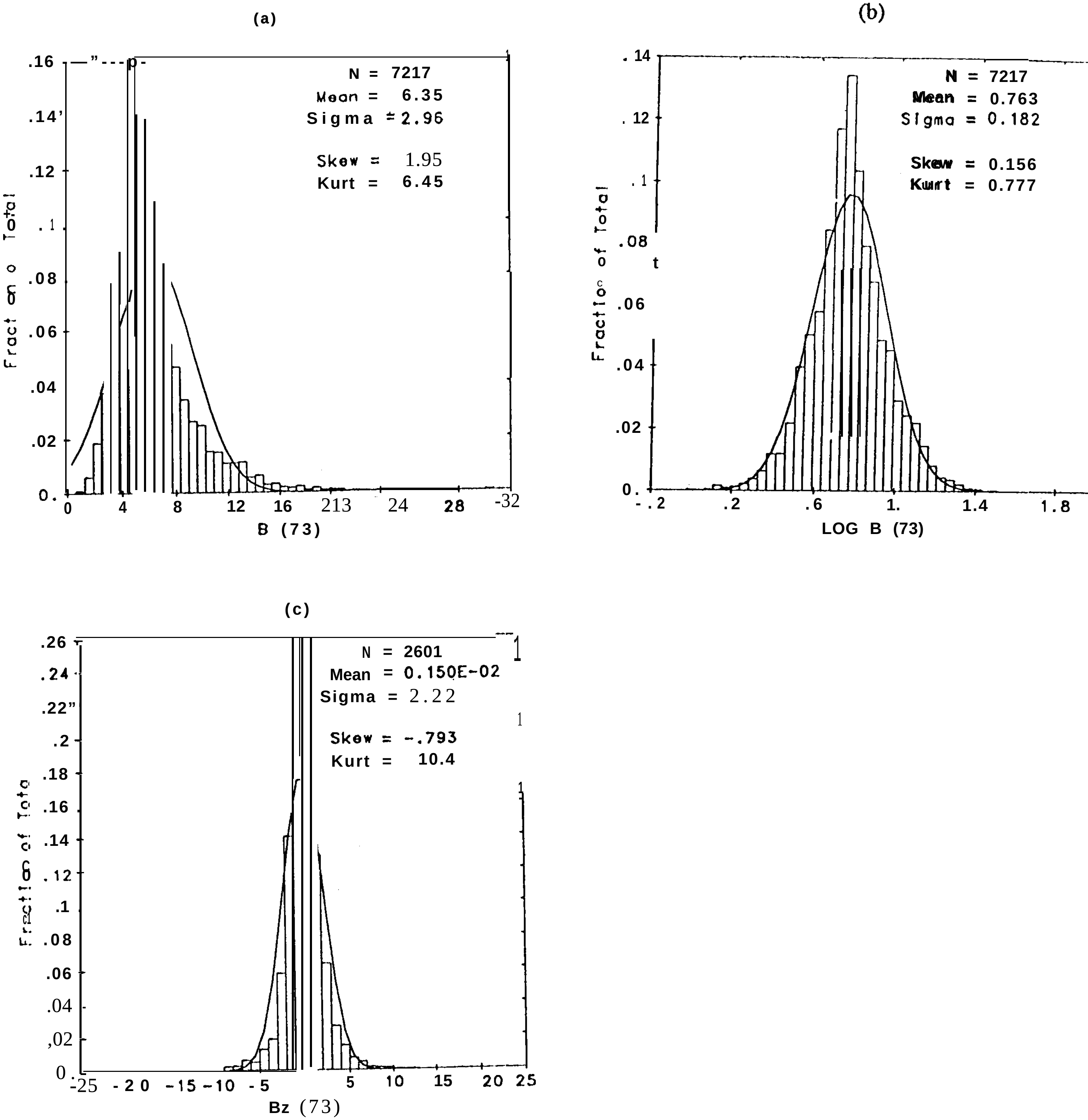
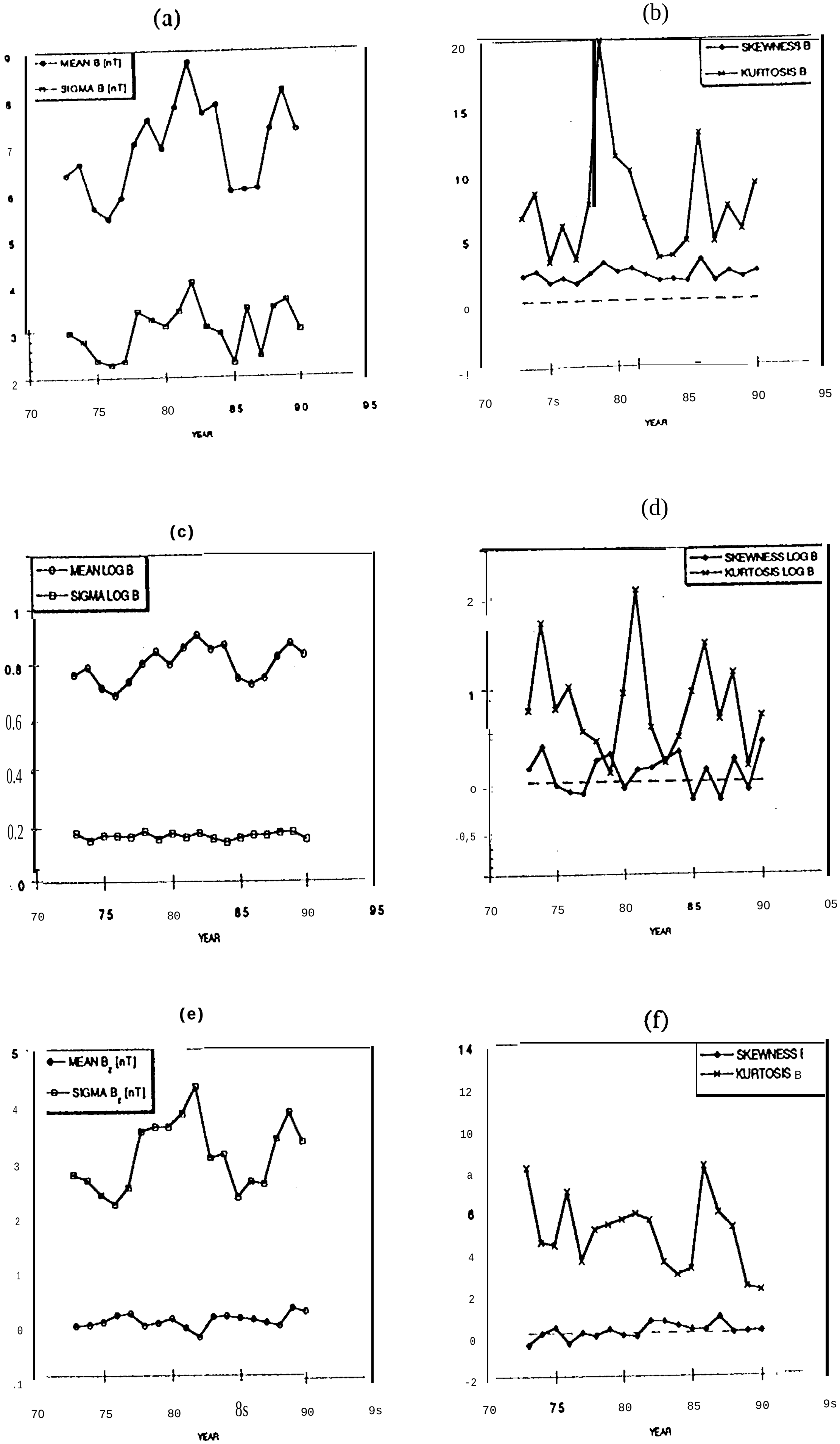

(a)
N = 7217
Mean = 6.35
Sigma = 2.96
Skew = 1.95
Kurt = 6.45
Fraction of Total
B (73)
(b)
N = 7217
Mean = 0.763
Sigma = 0.182
Skew = 0.156
Kurt = 0.777
Fraction of Total
LOG B (73)
(c)
N = 2601
Mean = 0.150E-02
Sigma = 2.22
Skew = -.793
Kurt = 10.4
Fraction of Total
Bz (73)

Fig 1



(a)
MEAN B (nT)
SIGMA B (nT)
YEAR
(b)
SKEWNESS B
KURTOSIS B
YEAR
(c)
MEAN LOG B
SIGMA LOG B
YEAR
(d)
SKEWNESS LOG B
KURTOSIS LOG B
YEAR
(e)
MEAN B_z [nT]
SIGMA B_z [nT]
YEAR
(f)
SKEWNESS
KURTOSIS B
YEAR

Fig. 2

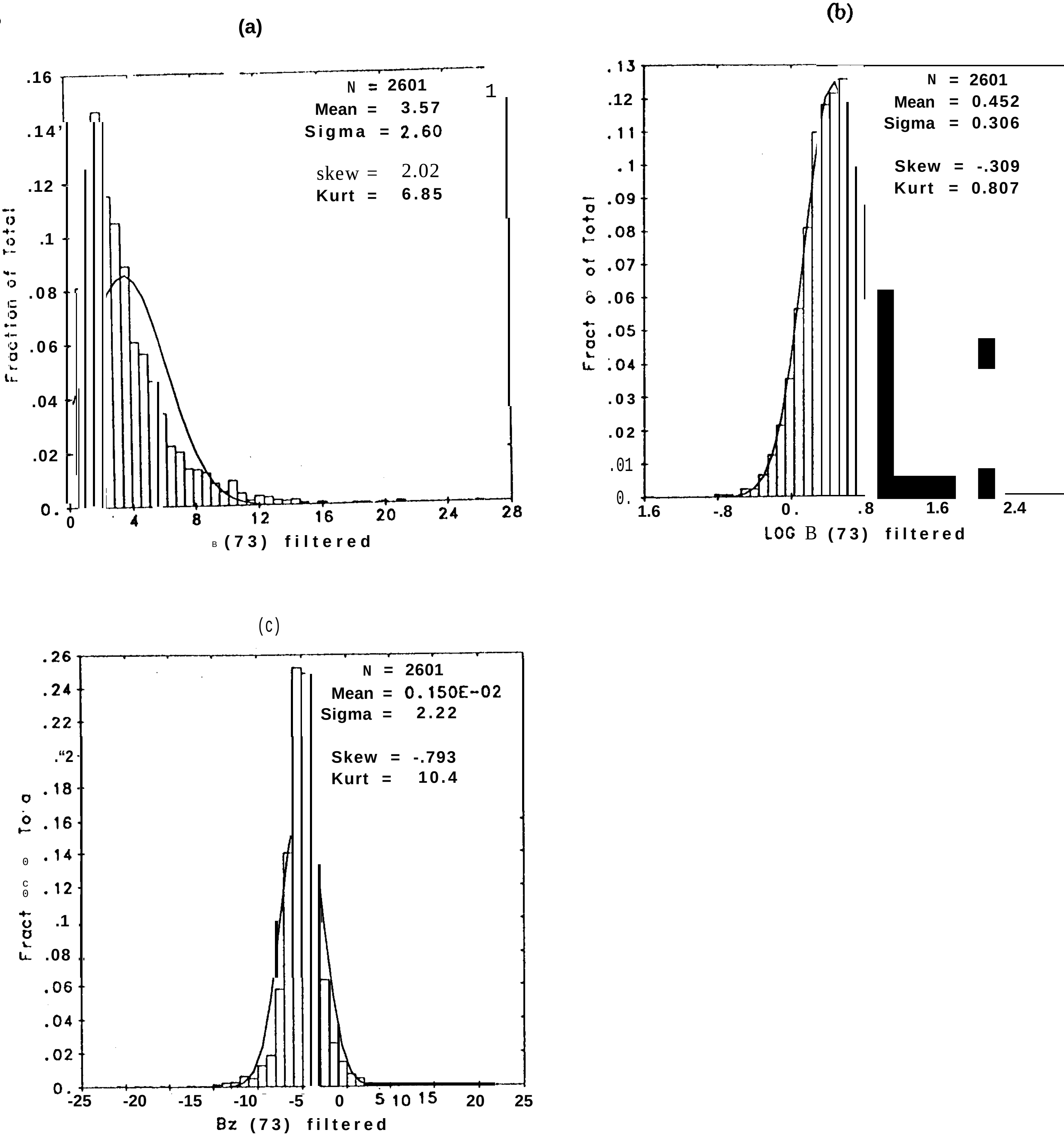

(a)
N = 2601
Mean = 3.57
Sigma = 2.60
skew = 2.02
Kurt = 6.85
Fraction of Total
B (73) filtered
(b)
N = 2601
Mean = 0.452
Sigma = 0.306
Skew = -.309
Kurt = 0.807
Fraction of Total
LOG B (73) filtered
(c)
N = 2601
Mean = 0.150E-02
Sigma = 2.22
Skew = -.793
Kurt = 10.4
Bz (73) filtered

Fig. 3

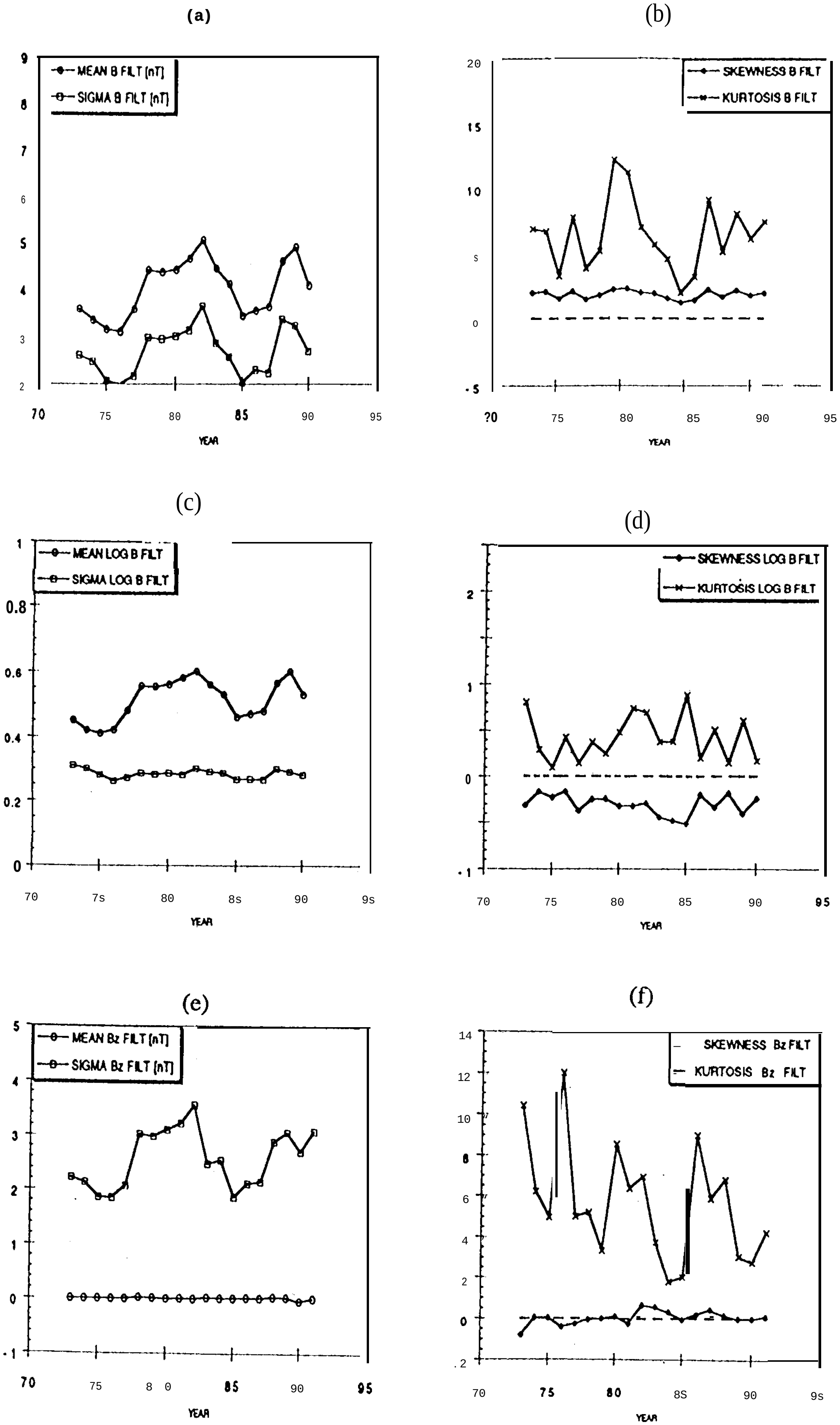

(a)
MEAN B FILT (nT)
SIGMA B FILT (nT)
YEAR
(b)
SKEWNESS B FILT
KURTOSIS B FILT
YEAR
(c)
MEAN LOG B FILT
SIGMA LOG B FILT
YEAR
(d)
SKEWNESS LOG B FILT
KURTOSIS LOG B FILT
YEAR
(e)
MEAN Bz FILT (nT)
SIGMA Bz FILT (nT)
YEAR
(f)
SKEWNESS Bz FILT
KURTOSIS Bz FILT
YEAR

Fig 4